Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2021

Subject Name : Advanced Real Analysis

Subject Code : 5SC03ARA1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 13/12/2021** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. True/False: Each finite measure is a σ - finite measure. **(01)**

b. Define: Measure **(02)**

c. Let $(X,\mathcal{A})$ be a measurable space if $\lambda \& \mu$ are two measure on $(X,\mathcal{A})$ with $\lambda \perp \mu$ and $\lambda << \mu$ then $\lambda = 0$. **(02)**

d. Let $(X,\mathcal{A})$ be any measurable space. Suppose μ is σ - finite measure on $(X,\mathcal{A})$ then μ is saturated. **(02)**

Q-2 **Attempt all questions** **(14)**

a. State and prove monotone convergence theorem. **(07)**

b. Define: Locally measurable set. Let S be a non-negative simple measurable function on a measure space $(X,\mathcal{A},\mu)$. If $\rho(E)=\int_E S\,d\mu$, $E\in\mathcal{A}$ then ρ is a measure on $(X,\mathcal{A})$. **(04)**

c. State and prove Beppo-Levi's theorem. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Lusin's theorem. **(09)**

b. Define: Complete measure space with example and prove that $(R,\mathcal{M},m)$ is a complete measure. Where R be a set of real numbers, $\mathcal{M}$ be the collection of Lebesgue measurable sets and *m* be Lebesgue measure. **(05)**

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Hahn-Decomposition theorem.	**(07)**
	b.	Let E be measurable set with $-\infty < \nu(E) < 0$ then E contains a negative set A with $\nu(A) < 0$.	**(05)**
	c.	Give an example of a set whose signed measure is negative but it may not be negative set.	**(02)**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Jordan Decomposition theorem.	**(09)**
	b.	State and prove Lebesgue Dominated Convergence theorem.	**(05)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	What do you mean by $\|f\|_{\infty}$?	**(01)**
	b.	Define: Baire measure, Compact	**(02)**
	c.	State Holder's inequality	**(02)**
	d.	Give an example of measures which are mutually singular.	**(02)**
Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Lebesgue Decomposition theorem.	**(10)**
	b.	If ν_1 and ν_2 are finite signed measure on $(X, \mathcal{A})$ then i) $\lvert\alpha\nu_1\rvert = \lvert\alpha\rvert\lvert\nu_1\rvert$ and ii) $\lvert\nu_1+\nu_2\rvert \le \lvert\nu_1\rvert + \lvert\nu_2\rvert$	**(04)**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Radon-Nikodym theorem.	**(08)**
	b.	State and prove Minkowski's inequality and also show that when it becomes equality.	**(06)**
Q-6		**Attempt all questions**	**(14)**
	a.	Prove that L^p spaces are complete spaces.	**(08)**
	b.	State and prove Density theorem.	**(06)**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove Caratheodory theorem.	**(08)**
	b.	Define Radon Nikodym derivative and write chain rule for it.	**(03)**
	c.	State Tonelli's theorem.	**(03)**